MET 4501 Fall 2024: Practice In-Class Quiz 5 Name: SOLUTION

INSTRUCTIONS:

This quiz is open-book and open-note, and you may work with your classmates. Please answer all questions and show all of your work.

GIVEN:

A spring inside a ballpoint pen fails catastrophically during normal use. The spring breaks and ejects violently from the pen, hitting the user in the eye, causing significant injury. We've been asked to investigate the incident to determine if this failure is due to defective design, inadequate testing or quality control, a material defect, or something else.

The helical compression spring, as designed, is made of no. 6 music wire. The outside coil diameter of the spring is 4.5 mm.

The ends are squared and there are 14 ½ total turns. *Note that "closed" and squared" are synonyms.*

FIND:

If the spring material is yielded during normal use. We'll break that task down into the below steps:

a) The torsional yield strength of the wire, S_{sy}.
b) The static load corresponding to the yield strength, F_y.
c) The spring rate, k.
d) The deflection that would be caused by the load in (b), y_y.
e) The solid length of the spring, L_s.
f) What length should the spring be to ensure that when it is compressed solid and then released, there will be no permanent change in the free length?
g) Given the length found in (f), is buckling a possibility?
h) The pitch of the body coil, p.

BONUS:

With a pair of calibrated calipers, measure the length of a spring inside an exemplar pen. Is the free length appropriate?

(a) THE STRENGTH OF SPRING WIRE DEPENDS ON THE WIRE SIZE, d. FROM TABLE A-28 IN SHIGLEY:

Table A–28 Decimal Equivalents of Wire and Sheet-Metal Gauges* (All Sizes Are Given in Inches)

Name of Gauge:	American or Brown & Sharpe	Birmingham or Stubs Iron Wire	United States Standard†	Manufacturers Standard	Steel Wire or Washburn & Moen	Music Wire	Stubs Steel Wire	Twist Drill
Principal Use:	Nonferrous Sheet, Wire, and Rod	Tubing, Ferrous Strip, Flat Wire, and Spring Steel	Ferrous Sheet and Plate, 480 lbf/ft³	Ferrous Sheet	Ferrous Wire Except Music Wire	Music Wire	Steel Drill Rod	Twist Drills and Drill Steel
7/0			0.500		0.490			
6/0	0.580 0		0.468 75		0.461 5	0.004		
5/0	0.516 5		0.437 5		0.430 5	0.005		
4/0	0.460 0	0.454	0.406 25		0.393 8	0.006		
3/0	0.409 6	0.425	0.375		0.362 5	0.007		
2/0	0.364 8	0.380	0.343 75		0.331 0	0.008		
0	0.324 9	0.340	0.312 5		0.306 5	0.009		
1	0.289 3	0.300	0.281 25		0.283 0	0.010	0.227	0.228 0
2	0.257 6	0.284	0.265 625		0.262 5	0.011	0.219	0.221 0
3	0.229 4	0.259	0.25	0.239 1	0.243 7	0.012	0.212	0.213 0
4	0.204 3	0.238	0.234 375	0.224 2	0.225 3	0.013	0.207	0.209 0
5	0.181 9	0.220	0.218 75	0.209 2	0.207 0	0.014	0.204	0.205 5
6	0.162 0	0.203	0.203 125	0.194 3	0.192 0	$\boxed{0.016}$	0.201	0.204 0
7	0.144 3	0.180	0.187 5	0.179 3	0.177 0	0.018	0.199	0.201 0
8	0.128 5	0.165	0.171 875	0.164 4	0.162 0	0.020	0.197	0.199 0
9	0.114 4	0.148	0.156 25	0.149 5	0.148 3	0.022	0.194	0.196 0
10	0.101 9	0.134	0.140 625	0.134 5	0.135 0	0.024	0.191	0.193 5
11	0.090 74	0.120	0.125	0.119 6	0.120 5	0.026	0.188	0.191 0
12	0.080 81	0.109	0.109 357	0.104 6	0.105 5	0.029	0.185	0.189 0
13	0.071 96	0.095	0.093 75	0.089 7	0.091 5	0.031	0.182	0.185 0
14	0.064 08	0.083	0.078 125	0.074 7	0.080 0	0.033	0.180	0.182 0
15	0.057 07	0.072	0.070 312 5	0.067 3	0.072 0	0.035	0.178	0.180 0
16	0.050 82	0.065	0.062 5	0.059 8	0.062 5	0.037	0.175	0.177 0
17	0.045 26	0.058	0.056 25	0.053 8	0.054 0	0.039	0.172	0.173 0

(Continued)

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$d = 0.016 \text{ in} = 0.41 \text{ mm}$

FROM TABLE 10-4 IN SHIGLEY:

Material	ASTM No.	Exponent m	Diameter, in	A, kpsi · in^m	Diameter, mm	A, Mpa · mm^m	Relative Cost of Wire
Music wire*	A228	$\boxed{0.145}$	0.004 to 0.256	201	✓0.10 to 6.5	$\boxed{2211}$	2.6
OQ&T wire†	A229	0.187	0.020 to 0.500	147	0.5 to 12.7	1855	1.3
Hard-drawn wire‡	A227	0.190	0.028 to 0.500	140	0.7 to 12.7	1783	1.0
Chrome-vanadium wire§	A232	0.168	0.032 to 0.437	169	0.8 to 11.1	2005	3.1
Chrome-silicon wire‖	A401	0.108	0.063 to 0.375	202	1.6 to 9.5	1974	4.0
302 Stainless wire#	A313	0.146	0.013 to 0.10	169	0.3 to 2.5	1867	7.6 to 11
		0.263	0.10 to 0.20	128	2.5 to 5	2065	
		0.478	0.20 to 0.40	90	5 to 10	2911	
Phosphor-bronze wire**	B159	0	0.004 to 0.022	145	0.1 to 0.6	1000	8.0
		0.028	0.022 to 0.075	121	0.6 to 2	913	
		0.064	0.075 to 0.30	110	2 to 7.5	932	

$A = 2211 \text{ MPa} \cdot \text{mm}^m, \quad m = 0.145$

THEREFORE, THE ULTIMATE TENSILE STRENGTH IS:

$$S_{ut} = \frac{A}{d^m} = \frac{2211}{(0.41)^{0.145}} = 2516 \text{ MPa}$$

THEN, FROM TABLE 10-6,

	Maximum Percent of Tensile Strength	
Material	Before Set Removed (includes K_W or K_B)	After Set Removed (includes K_s)
Music wire and cold-drawn carbon steel	$\boxed{45}$	60 to 70
Hardened and tempered carbon and low-alloy steel	50	65 to 75
Austenitic stainless steels	35	55 to 65
Nonferrous alloys	35	55 to 65

$$S_{sy} = 0.45\, S_{ut} = 0.45\,(2516 \text{ MPa}) = \boxed{1132 \text{ MPa}}$$

(b) F_y?

WE KNOW THAT $\tau = K_B \dfrac{8FD}{\pi d^3}$

AND F_y IS THE FORCE REQUIRED FOR $\tau = S_{sy}$. SO, REARRANGING:

$$F_y = \frac{\pi d^3 S_{sy}}{8 K_B D}$$

WE KNOW $D_o = 4.5\,mm$, SO

$$D = D_o - d = 4.5 - 0.41\,mm = 4.09\,mm$$

THE BERGSTRÄSSER FACTOR IS

$$K_B = \frac{4C+2}{4C-3}$$

WHERE $C = D/d$ IS THE SPRING INDEX.

$$C = \frac{D}{d} = \frac{4.09}{0.41} = 10$$

$$K_B = \frac{4C+2}{4C-3} = \frac{4(10)+2}{4(10)-3} = 1.14$$

SO, THE FORCE TO CAUSE STATIC YIELD IS

$$F_y = \frac{\pi(0.41\,mm)^3(1132\,MPa)}{8(1.14)(4.09\,mm)} = \boxed{6.6\,N}$$

(c)

$$k \approx \frac{d^4 G}{8D^3 N_a}$$

N_a CAN BE FOUND USING TABLE 10-1:

Type of Spring Ends				
Term	Plain	Plain and Ground	Squared or Closed	Squared and Ground
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, L_0	$pN_a + d$	$P(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
Pitch, p	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$(L_0 - 3d)/N_a$	$(L_0 - 2d)/N_a$

WE'RE GIVEN $N_t = 14.5$, SO

$$N_t = N_a + 2 \Rightarrow N_a = N_t - 2$$
$$= 14.5 - 2 = 12.5$$

G CAN BE FOUND IN TABLE 10-5:

Material	Elastic Limit, Percent of S_{ut}		Diameter d, in	E		G	
	Tension	Torsion		Mpsi	GPa	Mpsi	GPa
Music wire A228	65 to 75	45 to 60	<0.032	29.5	203.4	12.0	82.7
			0.033 to 0.063	29.0	200	11.85	81.7
			0.064 to 0.125	28.5	196.5	11.75	81.0
			>0.125	28.0	193	11.6	80.0
HD spring A227	60 to 70	45 to 55	<0.032	28.8	198.6	11.7	80.7
			0.033 to 0.063	28.7	197.9	11.6	80.0
			0.064 to 0.125	28.6	197.2	11.5	79.3
			>0.125	28.5	196.5	11.4	78.6
Oil tempered A239	85 to 90	45 to 50		28.5	196.5	11.2	77.2
Valve spring A230	85 to 90	50 to 60		29.5	203.4	11.2	77.2

$$G = 82.7\ GPa = 82.7 \times 10^9\ Pa$$

$$k \approx \frac{d^4 G}{8D^3 N_a} = \frac{(0.41\ mm)^4 (82.7\ GPa)}{8(4.09\ mm)^3 (12.5)}$$

$$= \boxed{0.342\ N/mm}$$

(d) y_y?

$$F = ky$$

$$y_y = F_y / k = \frac{6.6\ N}{0.342\ N/mm} = \boxed{19.2\ mm}$$

(e) L_s? FROM TABLE 10-1:

Type of Spring Ends				
Term	**Plain**	**Plain and Ground**	**Squared or Closed**	**Squared and Ground**
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, L_0	$pN_a + d$	$P(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$\boxed{d(N_t + 1)}$	dN_t
Pitch, p	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$(L_0 - 3d)/N_a$	$(L_0 - 2d)/N_a$

$$L_s = d(N_t + 1) = (.41\ mm)(14.5 + 1) = \boxed{6.36\ mm}$$

(f) L_0 TO AVOID YIELD?

$$L_0 = L_s + y_y = 6.36 + 19.2 \text{ mm} = \boxed{25.6 \text{ mm}}$$

(g) IS BUCKLING POSSIBLE?

THE ABSOLUTE STABILITY CONDITION IS:

$$L_0 < \frac{\pi D}{\alpha}\left[\frac{2(E-G)}{2G+E}\right]$$

FROM TABLE 10-5, E=203.4 GPa AND G=82.7 GPa

WE LOOK IN TABLE 10-2 FOR α:

End Condition	Constant α
Spring supported between flat parallel surfaces (fixed ends)	$\boxed{0.5}$
One end supported by flat surface perpendicular to spring axis (fixed); other end pivoted (hinged)	0.707
Both ends pivoted (hinged)	1
One end clamped; other end free	2

*Ends supported by flat surfaces must be squared and ground.

$$L_0 < \frac{\pi(4.09 \text{ mm})}{0.5}\left[\frac{2(203.4-82.7)}{2(82.7)+203.4}\right]$$

$$L_0 < 16.82 \text{ mm}$$

$$\boxed{25.6 \nless 16.82}$$

BUCKLING POSSIBLE!

(h) FROM TABLE 10-1:

Type of Spring Ends				
Term	Plain	Plain and Ground	Squared or Closed	Squared and Ground
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, L_0	$pN_a + d$	$P(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
Pitch, p	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$\boxed{(L_0 - 3d)/N_a}$	$(L_0 - 2d)/N_a$

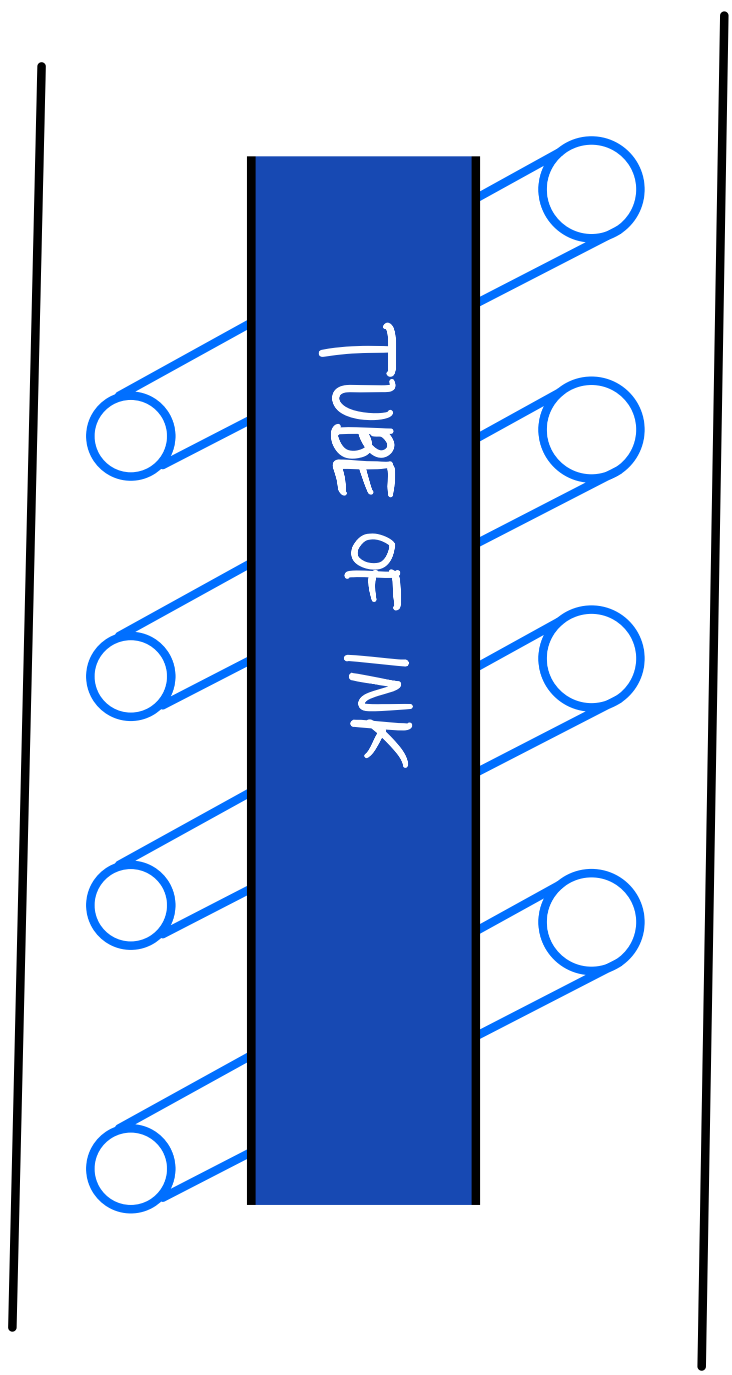
$$p = \frac{L_0 - 3d}{N_a} = \frac{25.6\,mm - 3(0.41\,mm)}{12.5}$$

$$\boxed{p = 1.95\,mm}$$

BONUS: ACTUAL $L_0 = 22$ mm FOR MY PEN, SO THE SPRING MIGHT BUCKLE.

WHAT PROTECTS AGAINST A FAILURE? WELL, THE SPRING IS RESTRAINED, WHEN PROPERLY INSTALLED, BY A

ROD (THE TUBE OF INK) AND BY BEING LOCATED IN A HOLE (INSIDE THE BODY OF THE PEN.)



IF INSTALLED AS SHOWN HERE, EVEN IF THE SPRING BUCKLES, IT WOULD CONTACT THE TUBE OF INK AND/OR THE BODY OF THE PEN.

SO, TO FAIL IN THE MANNER ALLEGED, IT SEEMS LIKE SOMETHING ELSE MUST BE AT PLAY.